Strategic Price Complexity in Retail Financial Markets

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Abstract

There is mounting empirical evidence to suggest that the law of one price is violated in retail financial markets: there is significant price dispersion even when products are homogeneous. Also, despite the large number of firms in the market, prices remain above marginal cost and may even rise as more firms enter. In a non-cooperative oligopoly pricing model, I show that these anomalies arise when firms add complexity to their price structures. Complexity increases the market power of the firms because it prevents some consumers from becoming knowledgable about prices in the market. As competition increases, firms tend to add more complexity to their prices as a best response, rather than make their disclosures more transparent. Because this may substantially decrease consumer surplus in these markets, such practices have important welfare implications.

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1 Introduction

Price formation in retail financial markets deviates from the predictions of standard price theory in several important ways. The law of one price is violated: significant price dispersion is present when goods and services are homogeneous. Despite the large number of firms in each market, prices do not converge to marginal cost. Even when new firms enter the industry, prices often do not decrease and may in fact rise. These pricing irregularities have been documented empirically in the markets for $S\&P$ Index funds (Hortacsu and Syverson 2004), money market funds (Christoffersen and Musto 2002), mutual funds (Bergstresser, Chalmers, and Tufano 2007), retail municipal bonds (Green, Hollifield, and Schürhoff 2007; Green 2007) credit cards (Ausubel 1991), conventional fixed-rate mortgages (Baye and Morgan 2001), life annuities (Mitchell, Poterba, Warshawsky, and Brown 1999), and term life insurance (Brown and Goolsbee 2002).

What is responsible for this seeming departure from classic microeconomics? The answer that I explore builds on the fifty year-old observation by Scitovsky (1950) that ignorance is a source of oligopoly power. Producers of retail financial products create ignorance by making their prices more complex, thereby gaining market power and the ability to increase industry profits. Clearly, many of the households who purchase retail financial products do not understand what they are buying and how much they are paying for these goods (e.g. Capon, Fitzsimons, and Prince 1996; Alexander, Jones, and Nigro 1998; Barber, Odean, and Zheng 2005; Agnew and Szykman 2005) and access to to financial advice does not appear to rectify this problem (Bergstresser, Chalmers, and Tufano 2007). Importantly, however, there appears to be a significant gap between investor knowledge about the financial instruments themselves and their understanding of industry fees. For example, in the NASD Literacy Survey (2003), 84% of market participants understood the relative riskiness of various bonds, but only 21% knew what a "no-load mutual fund" is. In fact, approximately one-third of the participants surveyed believed that the term no-load implies that there are no fees charged whatsoever.

In this paper, I consider the following important questions: How does complexity affect price formation in the market? How do firms optimally add complexity to their price structures to maximize profits? How do these optimal pricing policies change as industry competition increases? What is the effect of a professional financial advisor (i.e. an advice channel) on the complexity that is present in the market?

Financial institutions may add complexity to their prices in several ways. First, they can make it more difficult for households to become informed by partitioning prices into direct fees and indirect involuntary surcharges. This practice makes understanding prices more challenging as it places the responsibility on the consumer to appreciate all of the key price components and compute the actual price of the product. Second, complexity may be added when firms devise new technical language for their price disclosures. If firms in the industry use different methods of disclosure, this makes it more difficult for consumers to compare prices.

Complexity may also involve leaving out important information in a disclosure. This aspect makes it tougher for low-price firms to credibly signal their advantage because advertising is mechanism for signal jamming. For example, suppose a low-price mutual fund makes a statement that they have low management fees and no loads. A higher-priced fund can advertise that they have no management fees and no loads, even though they charge high 12b-1 fees and other indirect costs. Used in this way, complexity makes it harder for consumers to identify the best deals in the market.

In the paper, I begin by analyzing a two-stage pricing complexity game in which homogeneous firms produce an identical financial product and compete on price for market share. In the first period, firms simultaneously choose their prices (mutual fund fees, interest rates, etc.) and the complexity of their price structures. The complexity that one particular firm adds may increase the difficulty in evaluating their own price disclosure and comparing prices in the market, but does not affect the ability for consumers to evaluate the disclosures of competing firms. Based on the complexity choices of the firms, a fraction of consumers become informed about prices (*experts*), whereas the remainder remain *uninformed*. In the second period, the experts purchase the good from the low-priced firm, whereas uninformed consumers choose randomly from all of the firms.

In equilibrium, price dispersion arises because the firms compete strategically for market share from both types of consumers. This feature is also present in other models of search (e.g. Varian 1980; Stahl 1989), but arises here based on each firm's complexity decision (to be discussed shortly). The firm with the lowest price captures the entire share of expert consumers. All of the firms, however, receive some demand from the uninformed. The firms never charge marginal cost because they gain positive expected profits from sales to uninformed consumers. Also, it is impossible to have a one-price equilibrium in which all firms charge the same prices for their products. If they did so, one firm could undercut their competitors by a small amount and gain the entire market share from the expert consumers. So, equilibrium prices are strictly higher than marginal cost and there is always a non-degenerate distribution of prices (price dispersion).

Price complexity in the industry is determined through strategic interaction between the firms. In equilibrium, all firms enjoy a positive rent from having some degree of industry complexity and preventing some consumers from becoming informed. However, low-price firms desire less complexity than high-price firms. Since the low-price firms want consumers to know that they have the cheapest prices, they want the industry to be reasonably clear. Adding clarity allows them to undercut their rivals and gain market share. They do not want the industry to be too clear, however, as total clarity would erode industry rents altogether. In contrast, high-price firms desire more complexity. The more difficult the industry, the more uninformed consumers there will be and the more market share the high-price firms will receive. Industry confusion is the way high-price firms gain market share.

After deriving the equilibrium of the game, I consider how increased competition affects the way in which complexity evolves in the market. I first consider the effect that an advice channel has on complexity in the market. In the mutual fund industry, the advice channel is a system of brokers who offer investment advice to household investors. In the credit card, life insurance, and mortgage industries, price search engines assume the role of advisor. To study the effects induced by an advisor, I extend the original two-stage model to a four-stage game. The firms establish the industry as before, but in the second stage, advisors may market their information services to consumers. They charge consumers an advisory fee and incur an education cost of providing this service. In the third stage, uninformed consumers (non-experts) decide whether to pay for advice from the channel or whether to remain uninformed. In the fourth stage, consumers make their purchases based on the information that they have. In this version of the model, I show that the optimal complexity decision for the firms is isomorphic to that in the base model. In fact, the expected proportion of firms who choose to add higher complexity to their disclosures is identical to that in the base case. This result is important because it shows that the equilibrium complexity strategies for the firms remain robust even when an advisor provides information services in the market.

I then proceed to evaluate what happens to complexity when more firms compete for market share. I find that increased competition makes it more likely that firms make their price disclosures opaque. This result holds both when considering the base model and when an advice channel is present. When more firms compete for market share, the probability that they receive demand from the expert consumers decreases. As a best-response (i.e. to maximize expected profits), firms tend to increase complexity in order to maximize the revenues that they receive when they do not have the lowest price (when they lose the share of experts). In this case, increasing the fraction of uninformed consumers improves their expected profitability. The fact that more firms tend to add complexity when industry concentration decreases may induce a drop in the fraction of informed consumers, which in turn may increase producer surplus in the market. In this light, it is not surprising that Hortacsu and Syverson (2004) have shown that entry into the $S\&P$ index fund industry in 1995-1999 was associated with a rightward shift in the distribution of prices.

The model in this paper yields several novel empirical implications. For example, since the model implies that complexity is an important source of value for firms, changes in complexity should be positively correlated with firm profitability, *ceteris paribus*. Also, the model predicts that as competitive pressures rise in an industry (more producers enter the market), firms will respond by adding more complexity to their prices. This implies a negative correlation between industry concentration and price complexity. Though I do not test these predictions in this paper, they may be tested either cross-sectionally or with a time-series, using content analysis (e.g. Holsti 1969, Tetlock 2007) to quantify the amount of complexity present in various price disclosures.

This paper is of general economic interest as price dispersion has also been documented empirically in the markets for prescription drugs (Sorensen 2000), books (Clay, Krishnan, and Wolff 2001; Chevalier and Goolsbee 2003), and computer memory modules (Ellison and Ellison 2005). The paper also adds to an extensive literature on oligopoly pricing with consumer search (e.g. Diamond 1971; Salop and Stiglitz 1977; Varian 1980; Stahl 1989). In many existing models, the fraction of consumers who conduct incomplete search is exogenously given and the firms are unable to affect the search environment except through the prices they choose. Indeed, price dispersion arises in many of the models, but its source and its severity are determined by the exogenous parameters imposed in each model. In contrast, I provide a model in which firms endogenously affect the proportion of consumer types by altering the search environment. The complexity that firms add affects the proportion of consumers who become knowledgable, and in turn affects what they will pay for goods in the market. The "hide and seek" pricing model that I develop is new to this literature, and provides insight into the role of information in achieving industry price dispersion.

There is a growing related literature that evaluates how rational firms strategically set prices in response to the shortcomings of their consumer population. Heidhues and Koszegi (2005) study a monopolist's optimal pricing behavior when their consumers are loss averse. Perloff and Salop (1985) derive optimal pricing strategies given that all consumers make errors in calculating their value for the good. Gabaix, Laibson, and Li (2005) use extreme value theory to generalize the Perloff-Salop derivations. Gabaix and Laibson (2006) derive a "shrouded attributes equilibrium" in which prices are set and voluntary add-ons are chosen, all based on a given fraction of the consumers who are "myopic". The common theme in all of these papers is that the proportion of the consumers who are biased or less informed is exogenously given. In contrast, in this paper, this proportion evolves endogenously as the actions of the firms directly affect the proportion of experts who are present in the market.

The rest of the paper is organized as follows. In Section 2, I introduce the two-stage pricing complexity game. In Section 3, I characterize the strategic behavior of the firms and prove existence of a symmetric mixed strategy Nash equilibrium for the game. In Section 4, I analyze the four-stage game with an advice channel. Then, I proceed to characterize the effect that entry has on complexity. Section 5 concludes. The Appendix contains all of the proofs.

2 The Market for Retail Financial Products

Consider a market in which n firms, indexed by $j \in N = \{1, ..., n\}$, produce a homogeneous retail financial product. The product may be used by households to finance the purchase of consumption goods (for example, a credit card) or as an investment vehicle to maximize lifetime utility (for example, an index fund). The firms face zero marginal costs and have no capacity constraints. The only potential difference between the firms is the price that they charge and the complexity that they add to their price structures. Restricting the firms to produce an undifferentiated good is not just for technical convenience. Rather, if complexity in fee structures causes failure of competition and price dispersion in the market, adding heterogeneity in the product attributes will only make price dispersion more likely.

In the market, there is a unit mass of consumers M who each have unit demand for the retail

Figure 1: Pricing Complexity Game. At $t = 1$, n firms establish the industry structure. They each choose a price p_j for their product and a complexity level k_j for their price structure. At $t = 2$, the fraction of expert consumers purchase the good from the lowest price available and pay $p_{min} = min\{p_j\}_{j=1}^n$. Uninformed consumers buy from a randomly chosen firm and pay an expected price $\overline{p} = \frac{1}{n}\sum_{j=1}^n p_j$.

good. Every consumer i is risk-neutral and maximizes the expected payoff from their purchase. Their utility is given by

$$U_i = v - p_i$$

where v is the fundamental value of the product and p_i is the price that they pay. The value v is commonly known among the consumers and can be considered the monopoly price for the good. Because consumers receive v when they purchase the good from any firm, maximizing utility in this market is equivalent to minimizing the price that they pay.

Based on the actions of the firms, consumers are divided into two groups: financial experts (fraction μ) and uninformed consumers (fraction $1 - \mu$). That is, how complex the firms make their disclosures (to be specified shortly) endogenously affects the informedness of the consumer population. Experts are those consumers who become fully informed about the prices in the market and purchase the good at the lowest price available, $p_{min} = min\{p_j\}_{j=1}^n$. In contrast, uninformed consumers are those who remain illiterate about prices and purchase the good from a randomly chosen firm. As such, the probability that an uninformed consumer purchases the good from any one firm is $\frac{1}{n}$ and the expected price they will pay is $\overline{p} = \frac{1}{n}\sum_{j=1}^n p_j$. So that all uninformed consumers are rationally willing to participate in the market, I simplify the analysis and restrict the firms to choose prices $p_j \in [0, v]$.

The pricing complexity game is, therefore, a two-stage game (Figure 1), which proceeds as follows. In the first period ($t = 1$), the firms simultaneously set prices for the product and decide how complicated to make their price structure. Each firm chooses $p_j \in [0, v]$ and $k_j \in [\underline{k}, \overline{k}]$ where p_j is the actual (total) price that firm j charges and k_j is a measure of how difficult it is to sift through the price on all dimensions. I assume that the firms may choose any value for k_j in $[\underline{k}, \overline{k}]$ without paying a cost for doing so. The goal is to generate strategic choices for k_j that are independent of an exogenously imposed cost function. I define $\Sigma_j = [0, v] \times [\underline{k}, \overline{k}]$ to be the strategy space for firm j and $\sigma_j \in \Sigma_j$ to be firm j's (mixed) strategy over prices and complexity. In any Nash equilibrium, the strategies of the firms are given by the vector $\sigma^* = [\sigma_1^*, \ldots, \sigma_n^*]$.

The effect of complexity on the consumer population is captured mathematically as follows. The proportion of experts μ is determined by the multivariate map

$$\mu : [\underline{k}, \overline{k}]^n \rightarrow (0, 1) \tag{1}$$

such that $\mu(k_1, \ldots, k_n) \in C^2$, $\frac{\partial \mu}{\partial k_j} < 0$ for all j, and $\frac{\partial^2 \mu}{\partial k_j \partial k_\ell} = 0$ for all $j, \ell \neq j \in N$. The lower bound on μ, when all firms maximize their complexity and choose $\overline{k}$, is denoted by μ_{min}. Likewise, the upper bound on μ is denoted by μ_{max}. The set of restrictions that $\frac{\partial \mu}{\partial k_j} < 0$ for all j implies that as any one firm makes their price more difficult to evaluate, it makes the whole industry harder to analyze, and thereby lowers the fraction of experts. The last set of restrictions ($\frac{\partial^2 \mu}{\partial k_j \partial k_\ell} = 0$), however, implies that the complexity of one firm's price does not affect the inherent difficulty in evaluating a competing firm's offer.

The map in (1) captures the idea that complexity choices by individual firms not only makes it difficult to understand the price that is quoted by that particular firm, but also may make it more difficult to compare prices among firms. For example, suppose that one firm discloses their fees and uses particular technical language that a consumer must learn to interpret. A competing firm (say, Firm 2) may adopt the same language, or they may choose (or devise) an alternative form of disclosure, which the consumer must also learn. If Firm 2 devises an alternative form of disclosure, this will not change the ability of a consumer to understand Firm 1's price, but it does make it harder for the consumer to become fully knowledgable and compare prices. Therefore, while an individual firm's complexity choice may add difficulty to the overall task of becoming informed, it does not magnify the effect of other firms' complexity choices on the cost to become informed.

Once μ is realized, consumers make their purchases at $t = 2$. The firm (or firms) with the low price receives (their share of) the demand from the entire mass of experts as well a $(\frac{1}{n})$ share of the demand from uninformed consumers. The other firms receive only $\frac{1}{n}$ of the uninformed consumers' demand.

Partitioning consumers into two groups based on their knowledge of prices is standard in the search literature (e.g. Salop and Stiglitz (1977); Varian 1980; Stahl 1989) and has been referred to as an "all or nothing" search process or a "clearinghouse" search model (Baye, Morgan, and Scholten 2006). It is typical in this literature to consider either that the fraction of informed buyers is given exogenously by a constant μ, or evolves solely based on pricing behavior. This is where the analysis in this paper departs from standard approaches, in that I assume that the firms may influence the informedness of the consumer population by affecting the quality of information that they are given. As such, the reduced-form map in (1) is meant to capture the fact that individuals are adversely affected by complexity and the level of informedness in the population drops when prices become less transparent. An alternative specification might be to explicitly model a continuum of individual decision-makers (consumers) who choose optimally whether to become informed about the industry. As long as such a model does not allow for strategic interaction among the consumers, the implications from the model should not change qualitatively, though this conjecture remains unproven; I have found such a model to be limited by intractability and have chosen to pursue the more parsimonious model here. In Section 4, however, I do expand the analysis and consider the pricing and complexity dynamics in the market when uninformed consumers are given access to a financial advisor when making their purchase.

An inherent externality that arises in all or nothing search is that every firm's complexity choice affects the cost of the entire analysis (and therefore the fraction of experts) because it is mandatory to compare every firm's price to all of the others. This externality also arises in other types of search models (e.g. sequential search), however, and the results that follow are not unique to an all or nothing search process. It is also important to note that the model could be generalized to included partially-informed consumers. For technical simplicity, I do not include a fraction of consumers who use adaptive decision-making procedures (i.e., rules of thumb or heuristics) to narrow the field of choices (Payne, Bettman, and Johnson 1993). As will become clear in Section 3, only two consumer segments are required to generate price dispersion and the competitive effects described in the paper. Adding a third group, would segment the market further and make the model more complicated, but would not qualitatively change the results of the paper.

Finally, it is important to point out that I have not included advertising in this model. That is, firms may not randomly contact a fraction λ of the consumers at a cost, say $C(\lambda)$, to inform them about their price. Indeed, this has been explored previously by Robert and Stahl (1993), but not in a market where complexity is present. However, the results that follow would not change qualitatively as long as the cost of credibly advertising and educating the entire consumer population is exceedingly expensive. That is, as long as $C(1) = \infty$ (as in Robert and Stahl, 1993), a measure of consumers would still remain uninformed. Therefore, while advertising of this type does occur in reality and might make the industry more competitive, the inability of any one firm to educate all of the consumers in the population implies that the dynamics derived in this paper remain important.

3 Strategic Pricing and Complexity

In this section, I consider the firms' problem of creating an industry price structure. I pose the optimization problem faced by the firms and prove existence of a symmetric mixed-strategy Nash equilibrium for the game. I show that the equilibrium involves a positive price mark-up over marginal cost and that a symmetric equilibrium in pure strategies cannot exist. That is, there will always be price dispersion in this industry. Additionally, I derive the strategic complexity choices that the firms will employ when setting their prices.

Define J^* to be the set of firms who quote the lowest price in equilibrium. Let n_{j^*} be the number of firms in J^*, so that the n_{j^*} firms in J^* split the demand from the experts equally. Each firm j chooses p_j and k_j to maximize its expected profit, that is, they solve

$$\max_{\substack{p_j \in [0,v] \\ k_j \in [\underline{k}, \overline{k}]}} \quad \pi_j(p_j, k_j) = p_j Q_j \tag{2}$$

where the expected demand Q_j is calculated as

$$Q_j = \frac{\mu \ 1_{\{j \in J^*\}}}{n_{j^*}} + \frac{1-\mu}{n}.$$

Q_j is composed of two parts. The first expression represents the demand from the fraction μ of informed consumers. Firm j receives $\frac{1}{n_{j^*}}$ of this demand if they are one of the n_{j^*} firms that quote the lowest price in the industry. The second expression represents the expected demand from the fraction $1-\mu$ of uninformed consumers. As such, firm j's choice of k_j affects the proportion of experts μ and its choice of p_j affects whether they have the lowest price in the industry ($1_{\{j\in J^*\}}$).

The following proposition establishes the existence of a symmetric mixed-strategy Nash equilibrium in this pricing game and characterizes some of its properties.

Proposition 1. *(Existence and Characterization) In the pricing complexity game, there exists a symmetric mixed-strategy Nash equilibrium $\sigma^* = \{F^*(p), k^*(p)\}$ in which firms choose prices according to the distribution $F^*(p)$ and choose complexity according to the map*

$$k^*(p) = \begin{cases} \underline{k} & \text{if} \quad p < \hat{p} \\ \overline{k} & \text{if} \quad p > \hat{p} \\ k \in [\underline{k}, \overline{k}] & \text{if} \quad p = \hat{p}, \end{cases} \tag{3}$$

where

$$\hat{p} = {F^*}^{-1}\left(1 - \left[\frac{1}{n}\right]^{\frac{1}{n-1}}\right).$$

In equilibrium, the distribution function $F^*(p)$ is continuous and strictly increasing in p.

The ex ante probability that each firm chooses high complexity $\overline{k}$ is uniquely determined to be $\left[\frac{1}{n}\right]^{\frac{1}{n-1}}$. Additionally, the expected fraction of informed consumers $E[\mu]$ is also uniquely determined in equilibrium.

The proof of Proposition 1 is given in its entirety in the Appendix. The outline of the arguments used there is as follows. Inspecting (2), the payoff function for each firm j is continuous, except when its price is the lowest and is equal to at least one of its competitors. In this case, the firm may discontinuously increase (decrease) its payoff by lowering (raising) its price. It is possible to show, however, that each firm's payoff function is indeed weakly lower semi-continuous when its price is the lowest and equal to at least one of its competitors. Additionally, since the sum of the payoffs to all of the firms is a continuous function of any one firm's price, the pricing complexity game satisfies the conditions that are required for the existence of a symmetric mixed-strategy Nash equilibrium as outlined by Dasgupta and Maskin (1986). It is then possible to show that the optimal complexity choice for each firm only depends on its own price and the distribution of prices $F^*(p)$ that competing firms use to mix over prices, as defined in (3). The continuity and monotonicity of $F^*(p)$ follow from similar arguments as in Varian (1980). Finally, I conclude the proof by showing that that the ex ante probability of adding high complexity and the expected fraction of informed consumers is uniquely determined in equilibrium and only depend on n.

According to Proposition 1, prices are always dispersed in equilibrium since the distribution $F^*(p)$ has no mass points. This type of mixed-strategy is also present in other models of competitive pricing with consumer search (e.g. Varian 1980; Rosenthal 1980; Stahl 1989; Robert and Stahl 1993; Stahl 1996). As such, a one-price equilibrium is impossible and perfect competition does not arise in the market. To gain some intuition for this result, consider without loss of generality that all firms except firm j choose a pure pricing strategy such that $p_{-j} = p'$. Then,

$$\pi_j(p', k_j) = \frac{p'}{n}$$

for all $k_j \in [\underline{k}, \overline{k}]$. Since for $\epsilon > 0$ arbitrarily small

$$\pi_j(p' - \epsilon, \underline{k}) > \frac{p'}{n},$$

firm j has an incentive to undercut its competitors and lower its complexity to minimize any negative effects it has on the consumer population. The last inequality results from the fact that there is a market share (mass of experts) that firm j no longer has to share.

In several pricing models, this price cutting behavior causes a Bertrand Paradox, that is, prices equal marginal cost in equilibrium. However, a Bertrand Paradox does not arise in this market because marginal cost pricing is a dominated strategy. To see this, suppose that even one of firm j's competitors offers a price $p = 0$. Since

$$\pi_j(p_j, k_j) = \frac{p_j}{n}(1 - \mu) > \pi_j(0, k_j)$$

for any $p_j > 0$ and for all $k_j \in [\underline{k}, \overline{k}]$, firm j can earn positive expected profits by pricing its product strictly above its marginal cost. Based on this, Proposition 1 implies that there will exist a non-degenerate distribution of prices in equilibrium and an absence of marginal cost pricing.

According to Proposition 1, the firms choose their complexity given their draw from $F^*(p)$. Thus, it is not optimal for any firm j to mix over the entire strategy space Σ_j, that is, $k^*(\cdot)$ is a deterministic map based on p. The intuition behind this result is that each firm only needs to consider their expected relative price ranking, which is determined by the price that they choose and the distribution that competing firms use when they set their prices. For low-priced draws (below a threshold level $\hat{p}$), it is an optimal strategy to choose minimal complexity $\underline{k}$. For high-priced draws (above $\hat{p}$), it is optimal to choose $\overline{k}$. Intuitively, if a firm has a low price, they want consumers to be informed about it. The low-price firms will choose $\underline{k}$ to maximize the fraction μ of expert consumers. In contrast, if a firm has a relatively high price (above $\hat{p}$), they want consumers to be poorly informed. The high-price firms will choose $\overline{k}$ to minimize the the expert population. As long as $p \neq \hat{p}$, the firms choose k_j in a binary fashion as in (3). If indeed $p = \hat{p}$, firms are indifferent between any $k \in [\underline{k}, \overline{k}]$. However, since $F^*(p)$ is continuous and strictly monotonic, the event that $p = \hat{p}$ is of zero measure.

As such, the ex ante probability that a firm chooses high complexity to disclose their prices is set at

$$1 - F(\hat{p}) = \left[\frac{1}{n}\right]^{\frac{1}{n-1}},$$

which only depends on n. Since this probability is only a function of the number of firms, the expected informedness of the population ($E[\mu]$) is also pinned down by n. The cautious reader will probably have realized that Proposition 1 does not imply that there exists a unique equilibrium for the game, that is, that there may be more than one $F^*(p)$ that may exist in equilibrium. However, for any $F^*(p)$ that may exist, there will also exist a corresponding cutoff price $\hat{p}$ such that the probability that a firm chooses high complexity remains given by $\left[\frac{1}{n}\right]^{\frac{1}{n-1}}$. Therefore, even though uniqueness of the equilibrium remains unproven, the probability of making certain complexity choices and the expected fraction of experts is indeed pinned down uniquely.

In what follows, two interesting findings follow from this analysis. First, the results that are derived in Proposition 1 are robust to adding a professional advisor in the market. Second, since the tendency to add complexity to prices is completely dependent on the number of firms in the industry, we can evaluate how industry complexity will evolve based on how competitive the market becomes. These are the topics of the next section.

Purchase Channel by Households	Share	Share of Mutual Fund Assets	Percentage
Retirement Plan	48%	Advice Channel	55%
Advice Channel	37%	Retirement Plan	16%
Direct Sale	10%	Institutional	13%
Discount Broker/Supermarket	5%	Direct Sale	12%
		Supermarket	5%

Table 1: Primary Distribution Channel Used by Households. Left-hand column lists the percentage of households that purchase mutual funds though each channel. The right-hand column lists the fraction of mutual fund assets purchased through each channel. Source: Investment Company Institute July 2003.

4 Competition and Complexity

In this Section, I consider the effect of increasing competition on the complexity that arises in markets. I first analyze what happens when an advice channel markets its advice to uninformed consumers (non-experts). Then, I study how complexity changes when more firms compete for market share.

4.1 The Effect of an Advice Channel

In Section 2, I considered a model in which firms directly market their goods and services to consumers. As I pointed out, the model presented there is a reduced-form model of consumer search in which consumers rely on their own ability and resources to gain knowledge about the market and make an optimal purchase. In this section, I extend the analysis to consider how complexity evolves when a profit-maximizing advice channel is available to help uninformed consumers select which financial instruments to purchase. As I show, when the advice channel faces a fixed marginal cost of providing information to consumers, the equilibrium complexity choices by the firms in the industry are isomorphic to that derived in Section 2. Therefore, the analysis in Section 2 is robust to the presence of advisors.

In the mutual fund industry, the advice channel is a system of brokers who have expertise about the industry, but do not manage portfolios or trade directly. They offer advice and sell information to investors. In the United States, the advice channel accounts for 55% of the assets under management in the industry and 37% of households purchase mutual fund shares through the advice channel. Only 10% of mutual fund sales occur via a direct sale between mutual fund companies and individual investors (Table 1). In the mortgage, life insurance, and credit card industries, price search engines play the role of advisor. Websites established by intermediaries allow consumers to search for the best rates. Even though there is no human interaction at the website, the consumer depends on the search engine in the same way that they depend on a financial advisor.

Consider the four-period extension of the pricing complexity game in Figure 2. In the first stage, the firms choose p_j and k_j and μ is realized as in Section 2. In the second stage, homogeneous financial advisors observe the distribution of prices perfectly and choose whether to market their advice to uninformed consumers (if it is profitable to do so). If they indeed offer their services, they optimally choose a price w to maximize their profits from selling advice. The advisors pay a fixed cost $f > 0$ to set up shop and they pay an advisory cost A to educate each consumer that they serve. The advisory cost could be interpreted as the time-cost to educate others or the cost of developing learning materials.

In the third stage, expert consumers observe the entire distribution of prices perfectly, and are therefore able to identify the firm with the low price. In contrast, uninformed consumers observe some summary statistics about the industry (e.g. the average price in the market), but cannot use this information to rank order the firms in any way. Specifically, they are unable to identify the price that any particular firm quoted for the product. I denote by $\mathcal{I}$ the set of information that uninformed consumers have, which includes both these summary statistics as well as consistent beliefs about the equilibrium strategies of the firms. At $t = 3$ uninformed consumers decide whether to access the advice market (if it exists) and pay the fee w to identify the best deal in the market. In the last period, experts purchase the product from the lowest price firm in the market. Non-experts who paid for advice also get the best deal, whereas uneducated consumers purchase from a randomly chosen firm. As such, $\mathcal{I}$ only helps uninformed consumers to decide whether to pay for advice, but does not help them to narrow the field in choosing a firm if they remain uneducated. In any Nash equilibrium of the pricing complexity game with an advisor present, the strategies of the firms are given by the vector σ_A^*.

The game is solved by backward induction. At $t = 3$, uninformed consumers (non-experts) calculate the expected benefit of perfect information B and then decide whether to pay w for

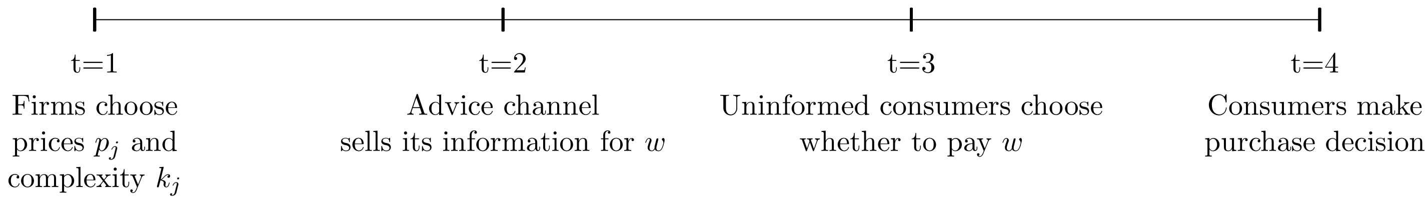

Figure 2: Pricing Complexity Game with an Advice Channel. At $t = 1$, the firms choose p_j and k_j. At $t = 2$, the advice channel offers retail information services to the consumers at price w. At $t = 3$, uninformed consumers choose whether to pay a cost w for advice or remain uninformed. At $t = 4$, consumers purchase, given the information they possess.

advice. They calculate B by considering the expected low price and the expected mean price, given their information set $\mathcal{I}$. As such, B is computed as

$$B = E[\bar{p}|\mathcal{I}] - E[p_{min}|\mathcal{I}].$$

When $w > B$, the advice channel does not earn any revenues since the price of advice exceeds the expected benefit of being informed. As long as $B > A$, it is possible for advisors to earn positive profits in this market once they enter. In what follows, I consider an advice channel in which the advisor has monopoly power, so that they maximize profits by setting $w = B$. Monopoly pricing may arise in the channel under three different settings. First, if the advice market is itself a search market in which all consumers face the same positive search cost for finding the best alternative, then all of the advisors will set monopoly prices in equilibrium (Diamond 1971). Second, if access to advice is frictionless and the advisors are homogeneous, then if more than one advisor enters the market, a Bertrand equilibrium will arise in the channel in which w will be set at the cost of serving customers, A. In this case, profits are negative since there are positive fixed set-up costs. Knowing this looking forward, once one advisor enters the market, it is a strictly dominated strategy for another one to follow suit. The third setting in which monopoly prices might arise is when advisors within the advice channel interact repeatedly as a going concern. In this case, repeated interaction gives the advisors the opportunity to set monopoly prices (and avoid the Bertrand Paradox) through implicit collusion (e.g. Abreu 1988).

Let us assume going forward that the fixed cost of entry f is very small compared to the potential margins that the advisor earns in the market. Specifically, assume that if $B > A$, then $B - A > f$

so that entry occurs if the marginal benefit of selling information is higher than the marginal cost. If this were not the case and f was exceedingly large, the problem at hand would reduce to the analysis in Section 2. Therefore, in period two, if $B > A$, an advisor provides information to all non-experts at a price $w = B$. In this case, the low-price firm (or firms) receives all of the demand in the market and firms with higher prices make zero profits. If $A > B$, however, then it is no longer profitable for the advice channel to provide information services and the expected equilibrium profits for all of the firms are the same as in Section 2.

At $t = 1$, because the firms cannot coordinate when they choose their prices and complexity, it is possible ex post (at $t = 2$) that either $A < B$ or $A > B$. If $A < B$, the low-price firm gets all of the industry demand. If in fact $A > B$, then all firms receive positive profits because some consumers remain uninformed. The firms take these two possibilities into account when choosing their optimal strategies at $t = 1$, given that their competitors are doing the same.

The following proposition establishes a symmetric mixed strategy equilibrium of the extended game and characterizes the strategic choices of the firms.

Proposition 2. *(Symmetric Mixed Strategy Equilibrium with Advice Channel) In the pricing complexity game with an advice channel, there exists a symmetric mixed-strategy Nash equilibrium* $\sigma_A^* = \{F_A^*(p), k_A^*(p)\}$ *in which firms choose prices according to* $F_A^*(p)$ *and choose complexity according to the map*

$$k_A^*(p) = \begin{cases} \underline{k} & \text{if} \quad p < \hat{p}_A \\ \overline{k} & \text{if} \quad p > \hat{p}_A \\ k \in [\underline{k}, \overline{k}] & \text{if} \quad p = \hat{p}_A, \end{cases} \tag{4}$$

where

$$\hat{p}_A = F_A^{*^{-1}}\left(1 - \left[\frac{1}{n}\right]^{\frac{1}{n-1}}\right).$$

In equilibrium, the distribution function $F_A^*(p)$ *is continuous and strictly increasing in* p.

The ex ante probability that each firm chooses high complexity $\overline{k}$ *is uniquely determined in equilibrium to be* $\left[\frac{1}{n}\right]^{\frac{1}{n-1}}$.

The logic involved in proving Proposition 2 follows same approach as in the proof of Proposition 1. Again, each firm's profit function is weakly lower semi-continuous when its price is the lowest and equal to at least one of its competitors. Given that the sum of the payoffs to all of the firms is a continuous function of any one firm's price, existence of a symmetric mixed-strategy Nash equilibrium is guaranteed by Dasgupta and Maskin (1986). The complexity strategy in (4) and the continuity and monotonicity of $F_A^*(p)$ follow using the same logic as in the proof of Proposition 1.

The importance that Proposition 2 has in this paper comes from the fact that the equilibrium choice of complexity has the same structure with and without a middleman advisor. It is striking that the ex ante expected amount of complexity that each firm adds is the same in either case. With or without an advice channel present, the probability that a firm adds high complexity only depends on n in exactly the same way.

This result probably arises for two reasons. First, the complexity choices of the firms do not affect the cost that the advisor incurs to provide information to consumers. If higher aggregate complexity indeed made the cost of education increase, then we might see firms add complexity to their prices to prevent advisors from profiting and disclosing information to uninformed consumers. Therefore, we can view the result in Proposition 2 as a best case scenario for consumers. That is, when the firms cannot affect the cost of education, the advisor's actions do not induce the firms to make their disclosures more transparent. If we did allow the firms to decrease the potential profitability for the advice channel due to increased education costs (secondary to increased complexity), it seems natural that firms might respond by making their pricing schedules even less transparent. While this prediction is only a conjecture, this strategy has been documented in non-financial markets. Indeed, Ellison and Ellison (2005) show that firms in the market for computer memory and central processing units (CPU's) increase complexity when there are price search engines in the market. By increasing the search frictions in the market, firms make it harder for the search engine to assist consumer search (rising A).

A second reason that complexity choices may be unchanged when an advisor is present is a lack of a coordination device among the firms. Since adding complexity is free and yet firms do not alter their strategies despite the fact that they face an ex post probability that they will receive zero demand, this represents a Prisoner's Dilemma that results from non-cooperative behavior. One potential coordinating device that is often present in these markets are incentive agreements between firms and advisors. In the mutual fund industry, front-end and back-end load fees are paid to the advice channel when shares in funds are purchased. In the mortgage, life insurance, and credit card industries, payments to the search engine occur in the form of sales commissions. With these types of incentives, advisors may find that their commission payments from the industry exceed the profits that they can generate from selling information to consumers. In this case, they will no longer provide complete information to consumers. In fact, recent work by Bergstresser, Chalmers, and Tufano (2007) shows that mutual funds recommended by an advisor tend to underperform those purchased through direct channels.

The robustness of the complexity strategies derived in (3) and (4), and their dependence on the number of firms leads us to a final analysis of the effect of industry concentration on complexity, which I consider next.

4.2 Entry, Complexity, and Financial Literacy

In what follows, I study how increasing competition affects the complexity that arises in the market. Given the model posed in Section 2, I show that as more firms compete in the industry, the probability that each firm adds complexity rises. The intuition is as follows. As more firms compete for the market share of the experts, any firm's chance of winning this business decreases. As a best-response, each firm tends to add more complexity, in an attempt to increase the fraction of consumers who are uninformed and increase their profits in the case that they do not win demand from the experts. Therefore, in aggregate the firms may use complexity to preserve industry rents in the face of higher competition.

The following proposition characterizes the strategic complexity choices that result when industry competition increases.

Proposition 3. *(The Effect of Entry on Complexity) In the pricing complexity game, the probability that a firm chooses high complexity ($\overline{k}$) is monotonically increasing in the number of firms n. In the limit as $n \to \infty$, all firms choose $\overline{k}$.*

The results in Proposition 3 can be appreciated analytically as follows. According to Proposition 1, the probability that a firm chooses $k_j = \overline{k}$ is

$$1 - F(\hat{p}) = \left[\frac{1}{n}\right]^{\frac{1}{n-1}}. \tag{5}$$

Since the right-hand side of (5) is increasing in n, each firm's probability of choosing high complexity is monotonically increasing in n. Likewise, taking the limit of $1 - F(\hat{p})$ as $n \rightarrow \infty$ yields

$$\lim_{n \to \infty} \left[\frac{1}{n}\right]^{\frac{1}{n-1}} \rightarrow 1.$$

Proposition 3 implies that a higher proportion of firms will add complexity to their prices when there is greater competition. Thus, we can draw a novel empirical prediction from the analysis: industry concentration and price complexity should be negatively correlated, *ceteris paribus*. As noted in the introduction, this prediction has yet to be tested. There does exist some indirect empirical evidence supporting this claim, however. We know from Hortacsu and Syverson (2004) that entry into the $S\&P$ index fund industry in 1995-1999 was associated with a rightward shift in the distribution of prices. Further, in their paper Hortacsu and Syverson estimate the search costs that investors faced to become informed during the period. They found that while these costs decreased for the bottom eighty-fifth percentile of the distribution, they increased for investors at the high end. Hortacsu and Syverson posit that this could be explained by the influx of novice investors. Another plausible explanation, however, might be that the market was becoming more challenging to analyze during this period, perhaps due to rising complexity. For example, if new participants in the market did indeed represent the top of the cost distribution and new participants in later years were not inherently less intelligent than new participants in prior years, increasing transparency in the market would imply that search costs should decrease uniformly over time. Since this does not appear to be the case empirically, it may imply that complexity may have been increasing during that period.

It is important to note that the analytical results in Proposition 3 are based on a map μ that is given exogenously in the model. As such, consumers are unable to respond strategically to the firms' complexity choices as the industry grows. For example, it may be a rational best-response for investors to form consumer-interest groups to coordinate their purchasing efforts (e.g. pool information). If the firms were to take this into account when setting their complexity levels, it might induce them make their disclosures more transparent. While this is not explicitly addressed by this model, the conclusions in Proposition 3 should be considered with some qualification.

It should also be pointed out that the relationship between competition and financial literacy remains equivocal in the model. That is, whereas complexity may increase in the face of higher competition, it is unclear whether financial literacy will suffer as a result. Analytically, this depends on the particular map μ that we consider. To gain intuition for this, let us consider two examples in which the fraction of experts in the market is a function of the average of the complexity choices for the firms in the market. Consider first that

$$\mu_1 = 1 - \frac{1}{n}\sum_{j=1}^{n} k_j, \tag{6}$$

where $0 < \underline{k} < \overline{k} < 1$. It is straightforward to verify that this map μ satisfies the conditions defined in Section 2. Further, consider that $\overline{k} - \underline{k} = \alpha$ such that $\alpha \in (0, 1)$. It follows then from (6) that $\mu_{max} - \mu_{min} = \alpha$.

Since each firm's complexity choice is binary and the probability of adding high complexity is $\left[\frac{1}{n}\right]^{\frac{1}{n-1}}$, the expected fraction of expert consumers in the market may be calculated as

$$E[\mu_1] = \mu_{max} - \sum_{m=0}^{n} \frac{n!}{m!(n-m)!}\frac{m\alpha}{n}\left(\left[\frac{1}{n}\right]^{\frac{1}{n-1}}\right)^{m}\left(1 - \left[\frac{1}{n}\right]^{\frac{1}{n-1}}\right)^{n-m} \tag{7}$$

where m is the number of firms who choose high complexity. By inspection of (7), $E[\mu_1]$ is computed as μ_{max} minus the expectation of a binomial random variable. The expected fraction of experts may then be computed as

$$E[\mu_1] = \mu_{max} - \frac{\alpha}{n}n\left[\frac{1}{n}\right]^{\frac{1}{n-1}} = \mu_{max} - \alpha\left[\frac{1}{n}\right]^{\frac{1}{n-1}},$$

which is decreasing in n. Therefore, as competition in the market becomes more fierce in this case, the expected number of informed consumers falls. In fact, as $n \to \infty$

$$\lim_{n\to\infty} E[\mu_1] \to \mu_{min}.$$

Now, consider in contrast that μ takes the form

$$\mu_2 = 1 - \frac{1}{n^2}\sum_{j=1}^{n} k_j, \tag{8}$$

where again $0 < \underline{k} < \overline{k} < 1$ and μ_2 satisfies the conditions defined in Section 2. As before, consider that $\overline{k} - \underline{k} = \alpha$ such that $\alpha \in (0,1)$. By construction, μ_2 is more sensitive to changes in n than μ_1, which might be the case if μ_2 captures the idea that the government adds educational initiatives as the industry grows in size or that consumer interest groups might be expected to arise as the industry evolves.

As before, each firm's complexity choice is binary and the probability of adding high complexity is $\left[\frac{1}{n}\right]^{\frac{1}{n-1}}$. The expected fraction of expert consumers in the market may be calculated as

$$E[\mu_2] = \mu_{max} - \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} \frac{m\alpha}{n^2} \left(\left[\frac{1}{n}\right]^{\frac{1}{n-1}} \right)^m \left(1 - \left[\frac{1}{n}\right]^{\frac{1}{n-1}} \right)^{n-m}. \tag{9}$$

As such, the expected fraction of experts is then computed as

$$E[\mu_2] = \mu_{max} - \frac{\alpha}{n^2} n \left[\frac{1}{n}\right]^{\frac{1}{n-1}} = \mu_{max} - \alpha \left[\frac{1}{n}\right]^{\frac{n}{n-1}},$$

which is increasing in n. Therefore, as competition increases, the expected number of informed consumers increases, despite the increased tendency for firms to add complexity. In this case, as $n \to \infty$

$$\lim_{n\to\infty} E[\mu_2] \to \mu_{max}.$$

The two examples highlight that financial literacy is a function of both the complexity choices of the firms, as well as factors outside of the model analyzed in this paper. Therefore, rising competition may have disparate effects on both consumer knowledge and welfare (through prices). Studying other such factors that are present in these markets (e.g. optimal regulation and the role of advisors) are the subject of future research, and would need to be considered in any empirical tests of the theory presented in this paper.

5 Conclusion

Purchasing a retail financial product requires effort. Because prices in the market are complex, consumers must pay a cost (time or money) to compare prices in the market. Some consumers gain sufficient expertise and get the best deal. Those with high search costs forego value, and often make purchases without knowing exactly what they are getting or how much they are paying. In fact, they may also be unaware that they are indeed over-paying (Choi, Laibson, and Madrian 2006). There is now a large literature that documents these findings (e.g. Capon, Fitzsimons, and Prince 1996; Alexander, Jones, and Nigro 1998; Barber, Odean, and Zheng 2005; Agnew and Szykman 2005) and this phenomenon may have substantial welfare implications (Campbell 2006; Calvet, Campbell, and Sodini 2007).

In this paper, I develop a model of pricing complexity in which firms compete on price for market share and strategically add complexity to preserve market power in the face of competitive pressures. The resulting equilibrium matches empirical observation: price dispersion persists even when goods are homogeneous and prices do not converge to marginal cost despite a large number of firms. The analysis in the paper has important social implications, given the large size of retail financial markets.

The paper also adds to an extensive literature on consumer search (e.g. Diamond 1971; Salop and Stiglitz 1977; Weitzman 1979; Varian 1980; Carlson and McAfee 1983; Stahl 1989). Whereas the search environment is constructed exogenously in most models in this literature, I consider a setting in which the firms may alter the quality of information transmission within the market, and affect the ability of consumers to become knowledgable about their purchases. The results that I derive capture several stylized features of retail financial markets, as well as markets in which other homogeneous products are sold (e.g. books). Therefore, the paper is of general economic interest as well.

I believe that this paper presents a plausible argument for considering complexity as an important determinant of price formation in retail financial markets. Given the large number of potential extensions of this analysis, this paper hopefully represents an important step toward greater understanding of the effect of complexity on security design, asset prices, and market structure.

Appendix

PROOF OF PROPOSITION 1:

Outline of Proof: The payoff function for each firm $j \in N$ is continuous, except when its price is the lowest and equal to at least one of its competitors. In this case, the firm may discontinuously increase (decrease) its payoff by lowering (raising) its price. According to Dasgupta and Maskin (1986), a symmetric mixed-strategy Nash equilibrium is guaranteed in this case as long as two conditions are satisfied:

i. The sum of the payoffs to all of the firms is upper semi-continuous.

ii. Each firm's payoff function is weakly lower semi-continuous at the points (actions) in the discontinuity set.

In what follows, I will show that these two conditions hold and by Theorem 6* in the appendix of Dasgupta and Maskin (1986), a symmetric mixed strategy Nash equilibrium exists for the game. Following this, I show that the optimal complexity choice for each firm completely depends on its own price and the distribution of prices $F^*(p)$ that competing firms use to mix over prices, as in (3). I then show that $F^*(p)$ is continuous and strictly increasing in equilibrium by following similar arguments as in Varian (1980). Finally, I conclude the proof by showing that that the ex ante probability of adding high complexity and the expected fraction of informed consumers is uniquely determined in equilibrium.

Proof: Since I prove existence using results derived by Dasgupta and Maskin (1986), I follow their notation. Let $A_j = [0, v] \times [\underline{k}, \overline{k}]$ be the action space for firm j and let $a_j \in A_j$ be a price-complexity pair in that space. As such, A_j is non-empty, compact and convex for all j. Define $A = \times_{j \in N} A_j$ and $a = (a_1, \ldots, a_n)$.

Let $U_j : A \to \mathbb{R}$ be defined as the profit function in (2). Define the set $A^*(j)$ by

$$A^*(j) = \{(a_1, \ldots, a_n) \in A | \exists i \neq j \ s.t. \ p_j = p_i\}$$

and the set $A^{**}(j) \subseteq A^*(j)$ by

$$A^{**}(j) = \{(a_1, \ldots, a_n) \in A | \exists i \neq j \ s.t. \ p_j = p_i = p_{min} > 0\}.$$

As such, the payoff function U_j is bounded and continuous, except over points $\bar{a} \in A^{**}(j)$.

The sum $\sum_{j \in N} U_j(a)$ is continuous since discontinuous shifts in demand from informed consumers between firms at points in $A^{**} = \times_{j \in N} A^{**}(j)$ occur as transfers between firms who have the same low price in the industry.

To prove the weak lower semi-continuity of $U_j(a_j, a_{-j})$, define B^2 to be the surface of the unit circle. Let $e = (e_1, e_2) \in B^2$ and let $\theta > 0$ be a positive number. Define $B_S^2 \subseteq B^2$ such that $B_S^2 = \{e | e_1 < 0\}$. Finally, define Ω to be the set of absolutely continuous distributions on B^2 such that for any point e that is not in B_S^2, $\omega(e) = 0$ for all $\omega \in \Omega$. Therefore, for any absolutely continuous measure $\omega \in \Omega$,

$$\int_{B^2} [\liminf_{\theta \to 0} U_j(a_j + \theta e, a_{-j}) d\omega(e)] = \int_{B_S^2} [\liminf_{\theta \to 0} U_j(a_j + \theta e, a_{-j}) d\omega(e)] \geq U_j(a_j, a_{-j}) \tag{A1}$$

for all $a_{-j} \in A_{-j}^{**}(a_j)$. Hence, $U_j(a)$ is weakly lower semi-continuous. Intuitively, since ω only places positive measure on points with prices less than p_{min}, the limit in the integral is strictly higher than the payoff the firm would receive if it had to share the demand from informed consumers with any of its competitors. Finally, (A1) holds with strict inequality when $a_j = a_i$ for all $i \in N/j$ (the so-called α property in Dasgupta and Maskin 1986).

Therefore, according to Theorem 6* of Dasgupta and Maskin (1986), since $\forall j \; A_j$ is non-empty, compact, and convex, $U_j : A \to \mathbb{R}$ is bounded and continuous, except over the set $A^{**}(j) \in A^*(j)$, $\sum_{j \in N} U_j(a)$ is continuous, $U_j(a)$ is weakly lower semi-continuous, and $U_j(a)$ satisfies the α property, there exists a symmetric mixed strategy Nash equilibrium of the game.

Remark 1. *The α property is not necessary for a symmetric mixed-strategy Nash equilibrium to exist, but does imply that $\forall \bar{a}_j \in A^{**}(j)$, the probability that firm j plays $\bar{a}_j$ is a measure zero event.*

Now we can characterize the equilibrium choice of complexity. Define $\Pi_j\Big(p_j, k_j | \sigma_{-j}\Big)$ as the expected profit for firm j when it chooses p_j and k_j, given the symmetric mixed strategies of the other firms σ_{-j}. Further, let us define two other functions:

$$\Gamma(p_j, k_j, \sigma_{-j}) \equiv E_{\sigma_{-j}}[\mu | p_j, k_j]$$

$$\Phi(p_j, k_j, \sigma_{-j}) \equiv E_{\sigma_{-j}}[\mu | p_j, k_j, p_j = p_{min}].$$

The function $\Gamma(p_j, k_j, \sigma_{-j})$ is the conditional expectation of μ given a choice of k_j and p_j for firm j and the strategies of the other firms. Likewise, $\Phi(p_j, k_j|\sigma_{-j})$ is the conditional expectation of μ, given firm j's choice of p_j and k_j, the strategies of the other firms, and that $p_j = p_{min}$. For clarity, I will use Γ and Φ in the rest of the proof to represent $\Gamma(p_j, k_j, \sigma_{-j})$ and $\Phi(p_j, k_j, \sigma_{-j})$, unless it is necessary to specify its arguments.

Suppose that all firms except for firm j use the strategy $\{F(p), H(k)\}$, where $F(\cdot)$ and $H(\cdot)$ are distributions that the firms use when they mix over each dimension of their action space. The expected profit for firm j is

$$\Pi(p_j, k_j|\sigma_{-j}) = p_j E_{\sigma_{-j}}\Big[\mu \, 1_{\{p_j = p_{min}\}} + \frac{1-\mu}{n}\Big],$$

which may be re-written as

$$\Pi(p_j, k_j|\sigma_{-j}) = p_j\Big[\Phi[1 - F(p_j)]^{n-1} + \frac{1-\Gamma}{n}\Big].$$

Differentiating $\Pi(p_j, k_j|\sigma_{-j})$ with respect to k_j yields

$$\frac{\partial \Pi}{\partial k_j} = p_j[1 - F(p_j)]^{n-1}\frac{\partial \Phi}{\partial k_j} - \frac{p_j}{n}\frac{\partial \Gamma}{\partial k_j}. \tag{A2}$$

Given that $\frac{\partial^2 \mu}{\partial k_j \partial k_\ell} = 0$, this implies that $\frac{\partial \Phi}{\partial k_j} = \frac{\partial \Gamma}{\partial k_j}$. That is, since the complexity choices of the other firms do not affect the magnitude with which changes in k_j affect the population of informed consumers, then $\frac{\partial \Phi}{\partial k_j}$ and $\frac{\partial \Gamma}{\partial k_j}$ are equal and only depend on firm j's choice of k_j (Note that this does not mean that $\Gamma = \Phi$).

Therefore, (A2) may be re-written as

$$\frac{\partial \Pi}{\partial k_j} = p_j\frac{\partial \Gamma}{\partial k_j}\Big[[1 - F(p)]^{n-1} - \frac{1}{n}\Big].$$

Since $\frac{\partial \Gamma}{\partial k} < 0$, if

$$[1 - F(p)]^{n-1} > \frac{1}{n},$$

we have $\frac{\partial \Pi}{\partial k} < 0$ and obtain the corner solution $k = \underline{k}$. This occurs when $p < \hat{p}$, where the threshold level $\hat{p}$ is

$$\hat{p} = F^{-1}\left(1 - \left[\frac{1}{n}\right]^{\frac{1}{n-1}}\right).$$

When $p > \hat{p}$, that is when

$$[1 - F(p)]^{n-1} < \frac{1}{n},$$

we have $\frac{\partial \Pi}{\partial k} > 0$ and obtain the other corner solution $k = \overline{k}$. Therefore, the equilibrium complexity choice for a firm only depend on a its choice of p. When $p \neq \hat{p}$, $k^*(p)$ is uniquely determined by (3), whereas when $p = \hat{p}$, the firm is indifferent between any $k \in [\underline{k}, \overline{k}]$.

Now, we can prove properties about $F^*(p)$.

i. Continuity: Suppose that there did exist a countable number of mass points in the distribution of $F^*(p)$. Then, we can find a mass point p' and an $\epsilon > 0$ such that $f^*(p') = a > 0$ and $f^*(p' - \epsilon) = 0$. Now consider a deviation by firm j to choose $\hat{F}(p)$ such that $\hat{f}(p') = 0$ and $\hat{f}(p' - \epsilon) = a$. Since $E[\Pi_j(p, k)]$ using $F^*(p)$ is strictly less than using $\hat{F}(p)$, this would be a profitable deviation. Therefore, in equilibrium, no mass points can exist.

ii. Strict monotonicity (Increasing): Suppose there exists an interval $[p_a, p_b]$ within $[0, v]$ such that $F(p_b) - F(p_a) = 0$. Then, for any $\hat{p}$ such that $p_a < \hat{p} < p_b$, $[1 - F(\hat{p})]^{n-1} = [1 - F(p_a)]^{n-1}$. Since $\hat{p}[1 - F(\hat{p})]^{n-1} > p_a[1 - F(p_a)]^{n-1}$ and $\hat{p}[1 - (1 - F(\hat{p}))^{n-1}] > p_a[1 - (1 - F(p_a))^{n-1}]$, then there exists a profitable deviation. Thus, $F(p_b) - F(p_a) \neq 0$ for any interval $[p_a, p_b]$ within $[0, v]$.

Remark 2. *As long as $\hat{p} \neq p_{min}$, $U_j(a_j, a_{-j})$ is continuous at $\hat{p}$ even though firm j's complexity choice is discontinuous at that point. Indeed, since each firm is indifferent between choosing any $k \in [\underline{k}, \overline{k}]$ when their price is $\hat{p}$, as long as $\hat{p} \neq p_{min}$, then $\lim_{p_j \xrightarrow{-} \hat{p}} U_j(a_j, a_{-j}) = U_j([\hat{p}, k(\hat{p})], a_{-j})$ and $\lim_{p_j \xrightarrow{+} \hat{p}} U_j(a_j, a_{-j}) = U_j([\hat{p}, k(\hat{p})], a_{-j})$. It follows then that each firm's profit function is continuous at $\hat{p}$ as long as this price is not the lowest in the market. In the case that $\hat{p} = p_{min}$, $[\hat{p}, k(\hat{p})] \in A^{**}(j)$. This implies that another equally credible approach to proving Proposition 1 would be to first prove that $k^*(p)$ takes the form in (3), and then prove existence of a mixed distribution $F^*(p)$ over which it is optimal for all firms to choose their price, given that their competitors are doing likewise.*

Proving existence of a symmetric mixed strategy Nash equilibrium reduces to a one-dimensional problem and Theorem 6 in Dasgupta and Maskin (1986) guarantees that such an $F^*(p)$ exists.

Now, we can consider the ex ante probability that a firm will add high complexity $\overline{k}$ to their price structure. For any $F^*(p)$ that may exist, there exists a corresponding threshold $\hat{p}$ such that with probability

$$1 - F^*(\hat{p}) = \left[\frac{1}{n}\right]^{\frac{1}{n-1}},$$

each firm will add high complexity to their prices. Since this probability only depends on n, it is uniquely determined in equilibrium. Based on this, the expected fraction of experts may be written as

$$E[\mu] = \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} \mu(m) \left(\left[\frac{1}{n}\right]^{\frac{1}{n-1}} \right)^m \left(1 - \left[\frac{1}{n}\right]^{\frac{1}{n-1}} \right)^{n-m}$$

where $\mu(m)$ is the fraction of informed consumers when m firms choose high complexity and $n-m$ firms choose low complexity. As such, $E[\mu]$ is uniquely determined in equilibrium by the probability of each firm choosing $\overline{k}$. ■

PROOF OF PROPOSITION 2:

The proof of Proposition 2 follows the same logic as in the proof of Proposition 1. The payoff function for each firm $j \in N$ is again continuous, except when its price is the lowest and equal to at least one of its competitors. In that case, the firm may discontinuously increase (decrease) its payoff by lowering (raising) its price. Showing that the sum of the payoffs to all of the firms is continuous and that each firm's payoff function is weakly lower semi-continuous at the points (actions) in the discontinuity set follows in the same way as in the proof of Proposition 1. Then, by Theorem 6* in the appendix of Dasgupta and Maskin (1986), a symmetric mixed strategy Nash equilibrium exists for the game.

To characterize the equilibrium complexity choices for the firms, define

$$\Psi(p_j, k_j, \sigma_{-j}) \equiv \Pr[A > B | p_j, k_j, \sigma_{-j}].$$

$$\Gamma_A(p_j, k_j, \sigma_{-j}) \equiv E_{\sigma_{-j}}[\mu | p_j, k_j, A > B]$$

$$\Phi_A(p_j, k_j, \sigma_{-j}) \equiv E_{\sigma_{-j}}[\mu | p_j, k_j, p_j = p_{min}, A > B].$$

The function $\Psi(p_j, k_j, \sigma_{-j})$ is the conditional probability that the costs of educating consumers exceeds the benefits of perfect information, given firm j's choices of p_j and k_j, and given the symmetric mixed strategies of the other firms. The function $\Gamma_A(p_j, k_j, \sigma_{-j})$ is the conditional expectation of μ given a choice of k_j and p_j for firm j, the strategies of the other firms, and information that $A > B$. Likewise, $\Phi(p_j, k_j | \sigma_{-j})$ is the conditional expectation of μ, given firm j's choice of p_j and k_j, the strategies of the other firms, the information that $A > B$, and that $p_j = p_{min}$. Again, for ease of notation, I will use Ψ, Γ_A, and Φ_A in the rest of the proof, unless it is necessary to specify the arguments of each function.

Because the firms cannot coordinate when they choose their prices and complexity, it is possible ex post that $A < B$. In this case, the low-price firm gets all of the industry demand. If in fact $A > B$, then all firms receive positive profits because some consumers remain uninformed. Suppose that all firms except for firm j use the strategy $\{F_A(p), H_A(k)\}$. The expected profit for firm j is

$$\Pi(p_j, k_j | \sigma_{-j}) = p_j \Big\{ \Psi \Big[\Phi_A (1 - F_A(p_j))^{n-1} + \frac{1 - \Gamma_A}{n} \Big] + (1 - \Psi)[1 - F_A(p_j)]^{n-1} \Big\}. \qquad \text{(A3)}$$

First order conditions with respect to k_j imply that

$$\frac{\partial \Pi}{\partial k_j} = p_j \left\{ \Psi \Big[\frac{\partial \Phi_A}{\partial k_j}(1-F_A(p_j))^{n-1} - \frac{1}{n}\frac{\partial \Gamma_A}{\partial k_j} \Big] - \frac{\partial \Psi}{\partial k_j}\Big[(1-\Phi_A)(1-F_A(p_j))^{n-1} - (1-\Gamma_A)\frac{1}{n}\Big] \right\}. \quad \text{(A4)}$$

Since $\frac{\partial \Psi}{\partial k_j} = 0$, then

$$\frac{\partial \Pi}{\partial k_j} = p_j \Psi [1 - F_A(p_j)]^{n-1} \frac{\partial \Phi_A}{\partial k_j} - \frac{p_j \Psi}{n}\frac{\partial \Gamma_A}{\partial k_j}. \quad \text{(A5)}$$

Given that $\frac{\partial^2 \mu}{\partial k_j \partial k_\ell} = 0$, this implies that $\frac{\partial \Phi_A}{\partial k_j} = \frac{\partial \Gamma_A}{\partial k_j}$. That is, since the complexity choices of the other firms do not affect the magnitude with which changes in k_j affect the population of informed consumers, then $\frac{\partial \Phi_A}{\partial k_j}$ and $\frac{\partial \Gamma_A}{\partial k_j}$ are equal and only depend on firm j's choice of k_j (Note that this does not mean that $\Gamma_A = \Phi_A$). Therefore, (A5) may be re-written as

$$\frac{\partial \Pi}{\partial k_j} = p_j \Psi \frac{\partial \Gamma_A}{\partial k_j}\left[[1 - F_A(p)]^{n-1} - \frac{1}{n} \right].$$

Since $\frac{\partial \Gamma_A}{\partial k} < 0$, if

$$[1 - F_A(p)]^{n-1} > \frac{1}{n},$$

we have $\frac{\partial \Pi}{\partial k} < 0$ and obtain the corner solution $k = \underline{k}$. This occurs when $p < \hat{p}_A$, where the threshold level $\hat{p}_A$ is

$$\hat{p}_A = F_A^{-1}\left(1 - \Big[\frac{1}{n}\Big]^{\frac{1}{n-1}}\right).$$

When $p > \hat{p}_A$, that is when

$$[1 - F_A(p)]^{n-1} < \frac{1}{n},$$

we have $\frac{\partial \Pi}{\partial k} > 0$ and obtain the other corner solution $k = \overline{k}$. Therefore, the equilibrium complexity choice for a firm only depend on a its choice of p. When $p \neq \hat{p}_A$, $k^*(p)$ is uniquely determined by (4), whereas when $p = \hat{p}_A$, the firm is indifferent between any $k \in [\underline{k}, \overline{k}]$.

The continuity and strict monotonicity of $F_A^*(p)$ and the uniqueness of the probability of choosing $\overline{k}$ follows from the same arguments in the proof of Proposition 1. ■

PROOF OF PROPOSITION 3: Taking the derivative of $\left[\frac{1}{n}\right]^{\frac{1}{n-1}}$ with respect to n yields

$$\left[\frac{1}{n}\right]^{\frac{1}{n-1}}\left[\frac{n\ln n-n+1}{n(n-1)^2}\right],$$

which is positive for all $n \geq 2$. Therefore, $1-F(\hat{p})$ is strictly increasing in n. Taking the limit

$$\lim_{n\to\infty}\left[\frac{1}{n}\right]^{\frac{1}{n-1}} \to 1.$$

■

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